Geometry
Assignment sheet

1. **<u>Read and study the geometry notes</u>**. There is a great deal of information here I know. If you have a graphing calculator or can have access to one it will be a great tool. I have also included a copy of the trig functions table to use if you do not have a calculator. I have attempted to explain how to use it.

2. I have included information from Module 13 Lessons 1 and 2. You should not need your workbooks. You may want hour notebooks but I think I have included all the information you need to complete these assignments.

<u>Module 13 Lesson</u> 1: Do the exercises at the end, 2 through 17

<u>Module 13 Lesson</u> 2: Do the exercises at the end, 3 through 16

<u>3. I have included Knowledge Checks for Module 13 Lessons 1 and 2. Please complete them as well</u>
These papers are for additional practice.

If you have questions please feel free to email me: dcobb@pisd.net

I am not sure at this time what days and times I will be here at school but please feel free to call me here as well.

I am not sure what procedure will be used for these assignments to be returned to me but I will be in contact with you or your parents though email.

Name: ____________________
Date: ____________________
Period: ____________________

Geometry Module 13 Notes

Key terms you will need to know.

Trigonometry
Side opposite or opposite side
Side adjacent or adjacent side
Short Leg
Long Leg
Hypotenuse
Tangent (tan)
Sine (sin)
Cosine (cos)
SOHCAHTOA – Sin – Opposite – Hypotenuse – Cos – Adjacent – Hypotenuse – Tan – Opposite - Adjacent

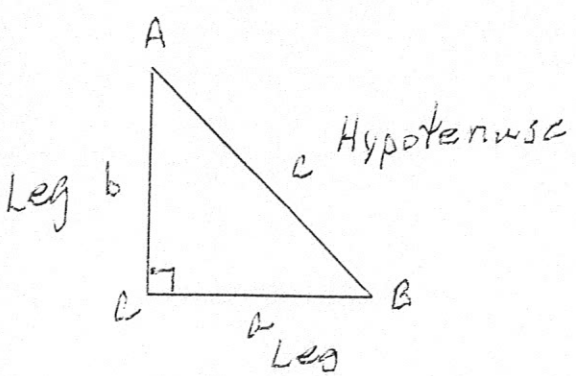
Tangent (tan) = $\dfrac{\text{length of the leg opposite of the angle}}{\text{Length of the leg adjacent of the angle}}$

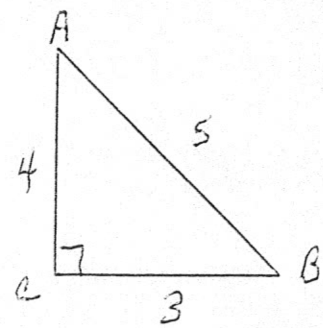
Sine (sin) = $\dfrac{\text{length of the leg opposite of the angle}}{\text{Length of the hypotenuse}}$

Cosine (cos) = $\dfrac{\text{length of the leg adjacent to the angle}}{\text{Length of the hypotenuse}}$

Pythagorean Theorem: $a^2 + b^2 = c^2$
Leg Squared + Leg Squared = Hypotenuse Squared

We have used this already in our work.
Remember that a "Pythagorean Triple", is a set of three numbers that make the Pythagorean Theorem true.
3 – 4- 5, 5 – 12 – 13, 7 – 24 – 25 are some examples we have discussed before.



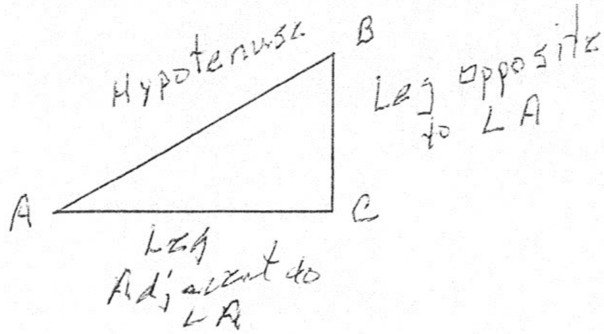


$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

Trigonometry is the study of measures of triangles and of the trigonometric functions and their applications. A trigonometric ratio is the ratio of two sides of a right triangle. Trigonometry helps us find angle measures and side lengths (distances). It is used a lot in science, engineering, art, video games and more.

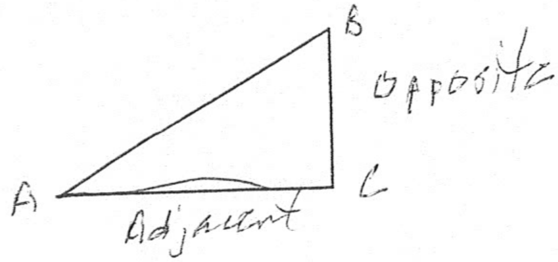
We will use three of the six trigonometric ratios. Sine, Cosine, and Tangent.

First we need to understand the meaning of the terms opposite side, adjacent side, and hypotenuse.
In the figure of the right triangle ABC below, segment AB is the hypotenuse, segment BC is the leg (side) opposite of angle A, and segment AC is the leg (side) adjacent to angle A. Remember, the hypotenuse is always the side opposite the right angle.

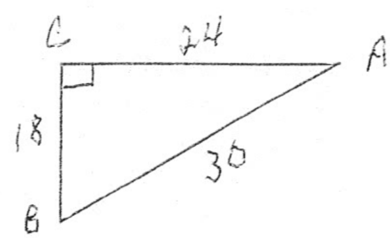


In Module 13 Lesson 1, we will focus on the tangent ratio, abbreviated tan. The tan ratio of angle A is the length of the opposite angle A over the length of the side adjacent to angle A.

$\tan \angle A = \frac{BC}{AC}$



To find the tangent of each acute angle of a right triangle we write the ratio (fraction) and then divide the bottom number into the top to get a decimal.

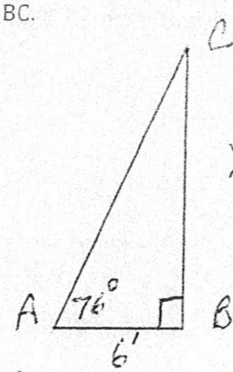


$$\tan A = \frac{\text{length of leg opposite of angle A}}{\text{length of leg adjacent of angle A}} = \frac{18}{24} = \frac{3}{4} = 0.75$$

$$\tan B = \frac{\text{length of leg opposite angle B}}{\text{length of leg adjacent angle B}} = \frac{24}{18} = \frac{4}{3} = 1.33$$

We now have to learn how to use these ratios and decimals to find the information we need to obtain. Most of the time this is the length of a side of a triangle or the measure of an angle of a triangle. There are two skills we must learn at this point. Under the circumstances we are operating I need to teach you how to use the calculator and how to use the Table of Trigonometric Values. Not all of you may have access to a calculator with trig functions on it. I have included a copy of the face of a TI=84 Plus calculator and a copy of the Table of Trigonometric Values. You may use which ever you wish to or which ever you must use. I will try and walk you through the steps to find the length of a side of right triangle and the measure of an angle of a right triangle.

Given the triangle below. It is a right triangle. We know the measure of angle A (76 degrees) and the length of segment AC 6 feet. We need to find the length of segment BC.



The first step is to write our equation using the tangent trig function.

$$\text{tan measure of angle A} = \frac{\text{length of leg opposite angle A}}{\text{length of leg adjacent angle A}}$$

$$\tan 76 = \frac{X}{6}$$

Then solve the equation for x.

$$6 \cdot \tan 76 = \frac{X}{\cancel{6}} \cdot \cancel{6}$$ Mult. both sides by 6.

$$6 \cdot \tan 76 = X$$

the tan key, is in the center of your calc.

Since we are finding a side length we use the tan button on the calculator. These are our key strokes:

6 times tan 76 enter. The calculator should read 24.0646856... Round your answer to the nearest tenth 24.1

The length of segment BC is 24.1 feet.

Olay, if you do not have a calculator you must use the table with the trig values. This is how it works. Go through the same steps to get your equation.

Then go to your table. Look under the m°∠A until you find the measure of 76 degrees. Then move over under tan A. The number should be 4.0108. Substitute this number into your equation. It should look like this.

$$6 \cdot \tan 76 = X$$
$$6 \cdot 4.0108 = X$$

Key strokes 6 times 4.0108 = 4.0648≈24.

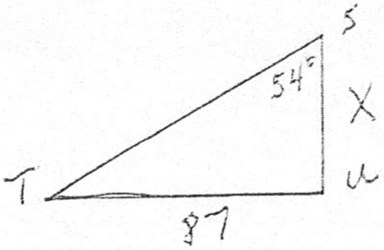
If you have a regular calculator.

Do the math. Your answer should be 24.1

TABLE OF TRIGONOMETRIC VALUES

m°∠A	sin A	cos A	tan A	m°∠A	sin A	cos A	tan A
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	~~77~~	0.9744	0.2250	~~4.3315~~
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713

We have the triangle below. We want to find the length of segment AC. This type of problem is the hardest. The algebra is more complicated. We have to add one algebraic step.



First, write our tan trig equation. tan measure angle S = $\frac{\text{length of side opposite}}{\text{length of side adjacent}}$

tan 54 degrees = $\frac{87}{x}$

Do the algebra. There has to be an additional step here. X is in the denominator. We must move it by multiplying each side by x. Then divide by tan 54. Then our key strokes are 87 divided by tan 54.

$$\tan 54 = \frac{87}{x}$$

$$x \cdot \tan 54 = \frac{87}{\cancel{x}} \cdot \cancel{x}$$

$$x \cdot \tan 54 = 87$$

$$\frac{x \cdot \cancel{\tan 54}}{\tan 54} = \frac{87}{\tan 54}$$

$$x = 87 \div \tan 54$$

$$x = 63.20919994$$

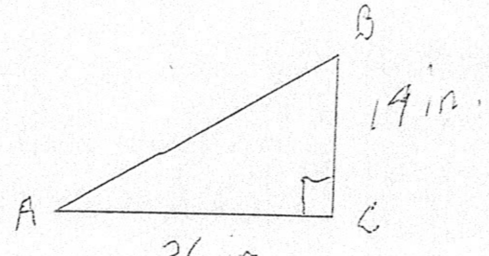
$$x = 63.2$$

Again, if you do not have a calculator with trig functions on it you must use the table. We need to find the tan of 54 degrees. Go to m $\angle$A. Go down to 54 degrees. The number should be 1.3764. put this number into your equation and do the math.

$$x = 87 \div 1.3764$$

$$x = 63.2$$

The next skill we must learn is to find the measure of an angle using the tangent function. If we have a right triangle and are given two side lengths but only one angle measure we can use trig to find the other angles. When we do this we use what is called the inverse tangent of an angle.



We want to find the measure of angle A. We know segment BC is opposite of angle A and segment AC is adjacent to angle A. We can use the tangent of angle A to find its measure.

First write the tangent of angle A equation. tan A = 19/36

Now if you have a graphing calculator we use the following key strokes.

First hit the 2nd button (the blue one) This puts us into the inverse mode.

Second type in 19 divided by 36 and then enter. Your calculator should read 27.82409638...

Round your answer to 27.8 degrees. This is the measure of angle A.

If you are using the table you have to divide 19 by 36. Use a regular calculator if you have one. You should get .5277777777... Round this to the fourth digit .5278. Go to your table. Look under tan ∠A. do down until you find a number close to .5278. You will be between .5095 and .5317. .5278 is closer to .5378 so we choose it. Now look to the left to the angle measure for ∠A. It is 28 degrees. That would be our answer.

TABLE OF TRIGONOMETRIC VALUES

m° ∠A	sin A	cos A	tan A	m° ∠A	sin A	cos A	tan A
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.50	0.8660	0.5774	75	0.9659	0.2588	3.7321

Lesson 2 takes the same concepts used with tangent in lesson 1 and applies them to the sine and cosine functions. We now have to determine which function to use. What information we are given will determine this.

Tangent is opposite side length over adjacent side length

Sine is opposite side length over the length of the hypotenuse.

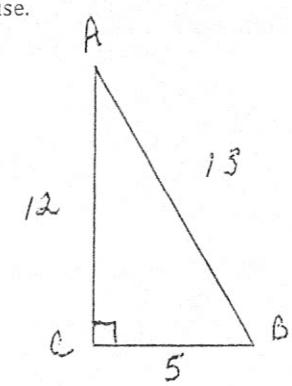
Cosine is the adjacent side length over the length of the hypotenuse.

Using this triangle we write each trig function as a ratio.

tan A = $5/12$ tan B = $12/5$

sin A = $5/13$ sin B = $12/13$

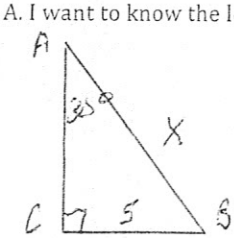
cos A = $12/13$ cos B = $5/13$



The task is to determine which function to use. Maybe this will help.

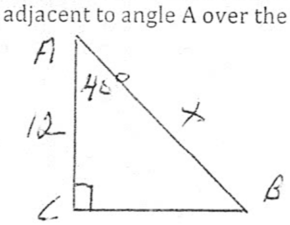
I know the measure of angle A. I know the length of the leg opposite of angle A. I want to know the length of the hypotenuse. I would use sin, because sin is opposite leg over hypotenuse.

$\sin A = \frac{5}{x}$ $\sin 30 = \frac{5}{x}$



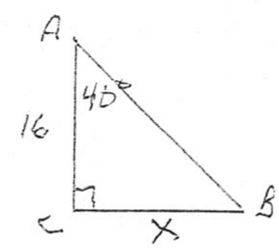
In this triangle I know the measure of angle A. I know the length of the leg adjacent to angle A. I want to know the length of the hypotenuse. I would use cos because cos is the length of the leg adjacent to angle A over the length of the hypotenuse.

$\cos 40 = \frac{12}{x}$



In this triangle I know the measure of angle A. I know the length of the leg adjacent to angle A. I want to know the length of the leg opposite to angle A. I would use tan, because tan is equal to the length of the leg opposite to angle A over the length of the leg adjacent to angle A.

$\tan 40 = \frac{x}{16}$



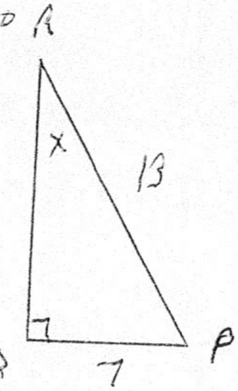
Once we determine which function to use it is a matter of applying the skills we have learned with the tangent function.

Find the measure of angle R in the following triangle. We know the length of the hypotenuse and the length of the leg opposite to angle R. We would use the sin function.

Since we want to find an angle measure we must use the inverse function. If we have a calculator we hit the 2nd button and this puts us in the inverse function mode. If we do not have a calculator and are using the chart we find the decimal by dividing 7 by 13. Then find this decimal on the chart under the sin function. This gives us our angle measure.

$\sin x = \frac{7}{13}$

2nd → sin 7 ÷ 13 → Enter $m\angle R = 32.6°$



$\sin x = \frac{7}{13}$

$\sin x = .5385$

Go to the table. Under sin A, .5385 is between .5299 and .5446

.5385 is closer to .5446 so we say $m\angle R = 33°$

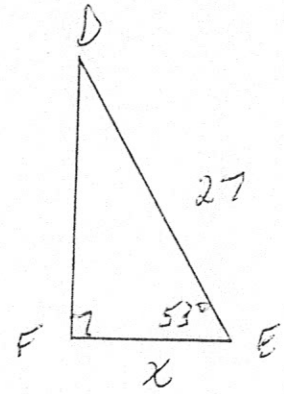
Find the length of segment EF in the following triangle. We know the measure of angle E and the length of the hypotenuse. We want to know the length of the leg adjacent to angle E. We would use the cos function.

Remember we must do the algebra first to get the x alone on one side of our equation. Then we can key into the calculator. If we use the table we find the cos of a 53 degree angle and multiply by 27.

$\cos 53 = \frac{x}{27}$

$27 \cdot \cos 53 = \frac{x}{\cancel{27}} \cdot \cancel{27}$

$27 \cdot \cos 53 = x$



Key strokes

27 → times → cos 53 Enter

$16.2 = x$

Chart

$27 \cdot .6018 = x$

$16.2 = x$

Always ask the question, what do I know and what do I need to find. Then apply this to which trig function you can use.

Name ________________________ Class ______________ Date __________

13.1 Tangent Ratio

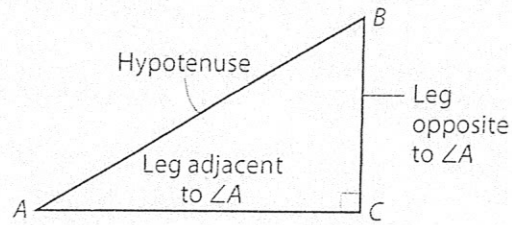
Resource Locker

Essential Question: How do you find the tangent ratio for an acute angle?

TEKS **G.9.A** Determine the lengths of sides and measures of angles in a right triangle by applying the...tangent to solve problems.

Explore Investigating a Ratio in a Right Triangle

In a given a right triangle, $\triangle ABC$, with a right angle at vertex C, there are three sides. The side **adjacent** to $\angle A$ is the leg that forms one side of $\angle A$. The side **opposite** $\angle A$ is the leg that does not form a side of $\angle A$. The side that connects the adjacent and opposite legs is the hypotenuse.

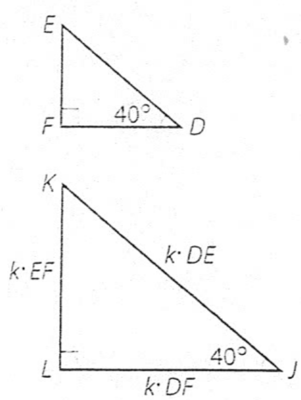


Explain 1 Finding the Tangent of an Angle

The ratio you calculated in the Explore section—the ratio of the length of the leg opposite an acute angle to the length of the leg adjacent to the acute angle—is called the *tangent* of an angle. The **tangent** of acute $\angle A$, written $\tan A$ is defined as follows:

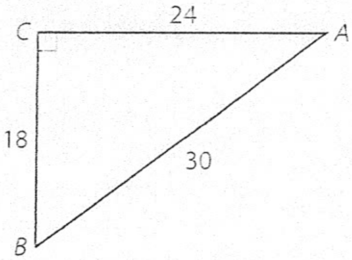
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

You can use what you know about similarity to show why the tangent of an angle is constant. By the AA Similarity Theorem, given $\angle D \cong \angle J$ and also $\angle F \cong \angle L$, then $\triangle DEF \sim \triangle JKL$. This means the lengths of the sides of $\triangle JKL$ are each the same multiple, k, of the lengths of the corresponding sides of $\triangle DEF$. Substituting into the tangent equation shows that the ratio of the length of the opposite leg to the length of the adjacent leg is always the same value for a given acute angle.



$$\tan 40^\circ = \overbrace{\frac{\text{leg opposite } \angle 40^\circ}{\text{leg adjacent to } \angle 40^\circ}}^{\text{tangent defined for specified angle}} = \overbrace{\frac{EF}{DF}}^{\triangle DEF} = \overbrace{\frac{KL}{JL} = \frac{k \cdot EF}{k \cdot DF}}^{\triangle JKL} = \frac{EF}{DF}$$

Example 1 Find the tangent of each specified angle. Write each ratio as a fraction and as a decimal rounded to the nearest hundredth.



Ⓐ $\angle A$

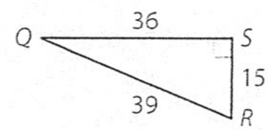
$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{18}{24} = \frac{3}{4} = 0.75$$

Ⓑ $\angle B$

$$\tan B = \frac{\text{length of leg Opposite } \angle B}{\text{length of leg Adjacent to } \angle B} = \frac{24}{18} = \frac{4}{3} \approx 1.33$$

Your Turn

Find the tangent of each specified angle. Write each ratio as a fraction and as a decimal rounded to the nearest hundredth.



5. $\angle Q$

$\tan \angle Q = \frac{15}{36} \approx 0.42$

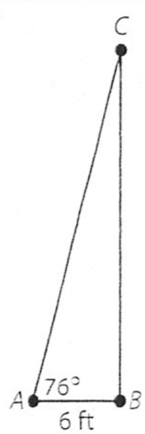
6. $\angle R$

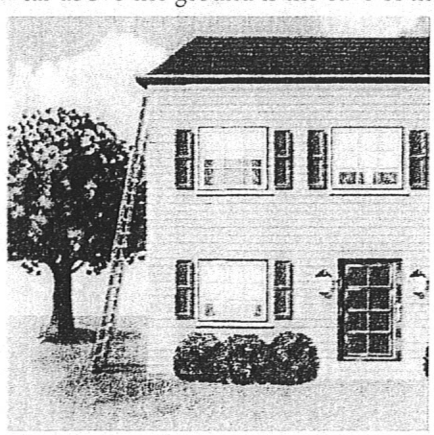
Explain 2 Finding a Side Length using Tangent

When you know the length of a leg of a right triangle and the measure of one of the acute angles, you can use the tangent to find the length of the other leg. This is especially useful in real-world problems.

Example 2 Apply the tangent ratio to find unknown lengths.

Ⓐ In order to meet safety guidelines, a roof contractor determines that she must place the base of her ladder 6 feet away from the house, making an angle of 76° with the ground. To the nearest tenth of a foot, how far above the ground is the eave of the roof?





Step 1 Write a tangent ratio that involves the unknown length.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{BA}$$

Step 2 Identify the given values and substitute into the tangent equation.

Given: $BA = 6$ ft and $m\angle A = 76°$

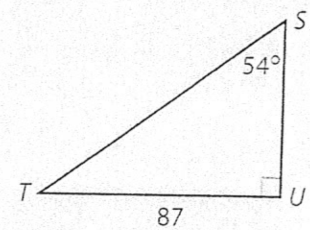
Substitute: $\tan 76° = \frac{BC}{6}$

Step 3 Solve for the unknown leg length. Be sure the calculator is in degree mode and do not round until the final step of the solution.

Multiply each side by 6.	$6 \cdot \tan 76° = \frac{6}{1} \cdot \frac{BC}{6}$
Use a calculator to find tan 76°.	$6 \cdot \tan 76° = BC$
Substitute this value in for tan 76°.	$6(4.010780934) = BC$
Multiply. Round to the nearest tenth.	$24.1 \approx BC$

So, the eave of the roof is about 24.1 feet above the ground.

(B) For right triangle $\triangle STU$, what is the length of the leg adjacent to $\angle S$?



Step 1 Write a tangent ratio that involves the unknown length.

$$\tan S = \frac{\text{length of leg opposite } \angle S}{\text{length of leg adjacent to } \angle S} = \frac{TU}{SU}$$

Step 2 Identify the given values and substitute into the tangent equation.

Given: $TU = 87$ and $m\angle S = 54°$

Substitute: $\tan 54° = \frac{87}{SU}$

Step 3 Solve for the unknown leg length.

Multiply both sides by SU, then divide both sides by 54°.	$SU = \frac{87}{\tan 54°}$
Use a calculator to find 54° and substitute.	$SU \approx \frac{87}{1.37638192}$
Divide. Round to the nearest tenth.	$SU \approx 63.2$

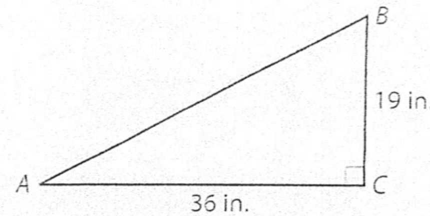
Your Turn

7. A ladder needs to reach the second story window, which is 10 feet above the ground, and make an angle with the ground of 70°. How far out from the building does the base of the ladder need to be positioned?

Explain 3 Finding an Angle Measure using Tangent

In the previous section you used a given angle measure and leg measure with the tangent ratio to solve for an unknown leg. What if you are given the leg measures and want to find the measures of the acute angles? If you know the tan A, read as "tangent of $\angle A$," then you can use the **tan^{-1} A**, read as "**inverse tangent of $\angle A$**," to find $m\angle A$. So, given an acute angle $\angle A$, if $\tan A = x$, then $\tan^{-1} x = m\angle A$.

Example 3 Find the measure of the indicated angle.
Round to the nearest degree.



Ⓐ What is m∠A?

Step 1 Write the tangent ratio for ∠A using the known values.	Step 2 Write the inverse tangent equation.	Step 3 Evaluate using a calculator and round as indicated.
$\tan A = \frac{19}{36}$	$\tan^{-1}\frac{19}{36} = m\angle A$	$m\angle A \approx 27.82409638 \approx 28°$

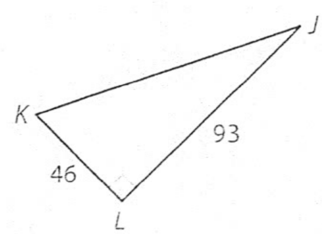
Ⓑ What is m∠B?

Step 1 Write the tangent ratio for ∠B using the known values.	Step 2 Write the inverse tangent equation.	Step 3 Evaluate using a calculator and round as indicated.
$\tan B = \frac{36}{19}$	$\tan^{-1}\frac{36}{19} = m\angle B$	$m\angle B \approx 62.17590362° \approx 62°$

Your Turn

$m\angle J = \tan^{-1}\frac{46}{93} \approx 26.31808814$

8. Find m∠J.

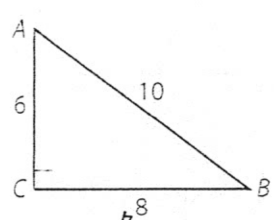


$m\angle J = 26°$

Evaluate: Homework and Practice

In each right triangle, find the tangent of each angle that is not the right angle.

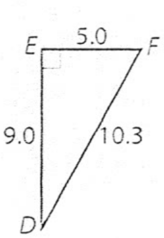
2.



$\tan\angle A = \frac{8}{6}$ $\tan\angle B = \frac{6}{8}$

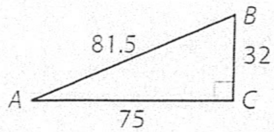
$\tan\angle A \approx 1.33$ $\tan\angle B \approx 0.75$

3.

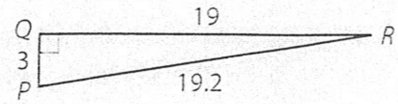


4

4.

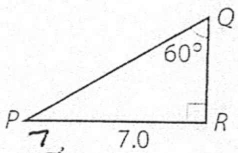


5.



Use the tangent to find the unknown side length.

9. Find *QR*.

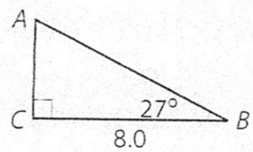


$\tan 60^\circ = \frac{7}{QR}$

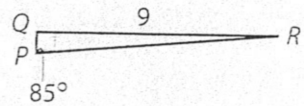
$QR = \frac{7}{\tan 60^\circ}$

$QR = 4$

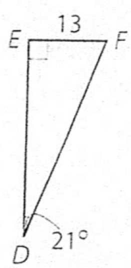
10. Find *AC*.



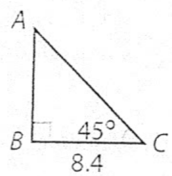
11. Find *PQ*.



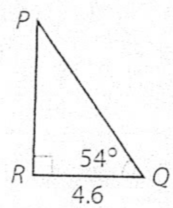
12. Find *DE*.



13. Find *AB*.

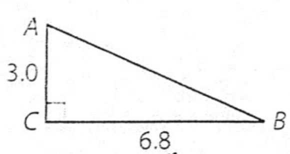


14. Find *PR*.



Find the measure of the angle specified for each triangle. Use the inverse tangent ($\tan^{-1}$) function of your calculator. Round your answer to the nearest degree.

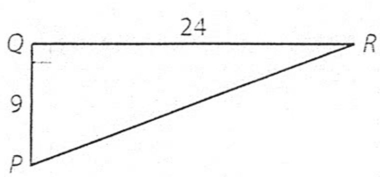
15. Find $\angle A$.



$\tan^{-1} \frac{6.8}{3} = m\angle A$

$m\angle A = 66^\circ$

16. Find $\angle R$.



17. Find $\angle B$.