Name ________________ Class ________ Date ________

13.2 Sine and Cosine Ratios

Essential Question: How can you use the sine and cosine ratios, and their inverses, in calculations involving right triangles?

Resource Locker

TEKS G.9.A Determine the lengths of sides and measures of angles in a right triangle by applying the ... sine, cosine ... to solve problems.

Explain 1 Finding the Sine and Cosine of an Angle

Trigonometric Ratios

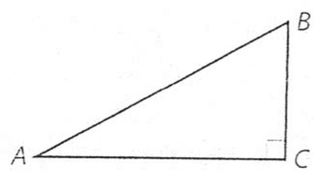
A **trigonometric ratio** is a ratio of two sides of a right triangle. You have already seen one trigonometric ratio, the tangent. There are two additional trigonometric ratios, the sine and the cosine, that involve the hypotenuse of a right triangle.

The **sine** of $\angle A$, written $\sin A$, is defined as follows:

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{BC}{AB}$$

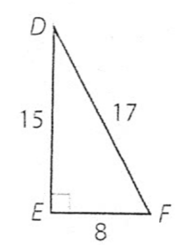
The **cosine** of $\angle A$, written $\cos A$, is defined as follows:

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AC}{AB}$$



You can use these definitions to calculate trigonometric ratios.

Example 1 Write sine and cosine of each angle as a fraction and as a decimal rounded to the nearest thousandth.



Ⓐ $\angle D$

$$\sin D = \frac{\text{length of leg opposite } \angle D}{\text{length of hypotenuse}} = \frac{EF}{DF} = \frac{8}{17} \approx 0.471$$

$$\cos D = \frac{\text{length of leg adjacent to } \angle D}{\text{length of hypotenuse}} = \frac{DE}{DF} = \frac{15}{17} \approx 0.882$$

Ⓑ $\angle F$

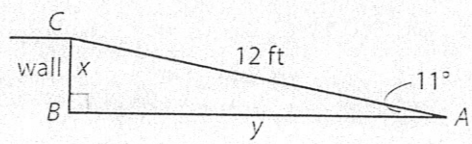
$$\sin F = \frac{\text{length of leg opposite to } \angle F}{\text{length of hypotenuse}} = \frac{DE}{DF} = \frac{15}{17} \approx 0.882$$

$$\cos F = \frac{\text{length of leg adjacent to } \angle F}{\text{length of hypotenuse}} = \frac{8}{17} \approx 0.471$$

Explain 3 Finding Side Lengths using Sine and Cosine

You can use sine and cosine to solve real-world problems.

Example 3 A 12-ft ramp is installed alongside some steps to provide wheelchair access to a library. The ramp makes an angle of 11° with the ground. Find each dimension, to the nearest tenth of a foot.



(A) Find the height x of the wall.

Use the definition of sine. $\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}} = \frac{AB}{AC}$

Substitute 11° for A, x for BC, and 12 for AC. $\sin 11° = \frac{x}{12}$

Multiply both sides by 12. $12\sin 11° = x$

Use a calculator to evaluate the expression. $x \approx 2.3$

So, the height of the wall is about 2.3 feet.

(B) Find the distance y that the ramp extends in front of the wall.

Use the definition of cosine. $\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}} = \frac{AB}{AC}$

Substitute ° for A, y for AB, and for AC. $\cos 11° = \frac{y}{12}$

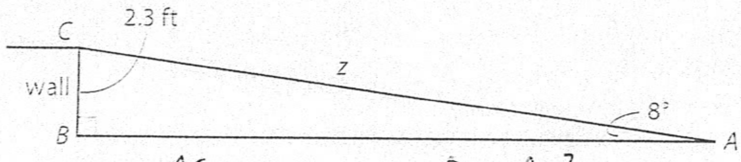
Multiply both sides by 12. $12 \cos 11° = y$

Use a calculator to evaluate the expression. $y \approx 2.3$

So, the ramp extends in front of the wall about 2.3 feet.

Your Turn

11. Suppose a new regulation states that the maximum angle of a ramp for wheelchairs is 8°. At least how long must the new ramp be? Round to the nearest tenth of a foot.



$\sin A = \frac{BC}{AC} \Rightarrow \sin 8^\circ = \frac{2.3}{z}$

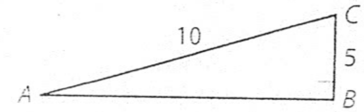
$z \approx 16.5$

This is the most difficult step. You have to multiply both sides by z and then divide both sides by sin 8.

Sometimes I tell students to just trade places with these two terms.

Explain 4 Finding Angle Measures using Sine and Cosine

In the triangle, $\sin A = \frac{5}{10} = \frac{1}{2}$. However, you already know that $\sin 30^\circ = \frac{1}{2}$. So you can conclude that $m\angle A = 30^\circ$, and write $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$.



Extending this idea, the **inverse trigonometric ratios** for sine and cosine are defined as follows:

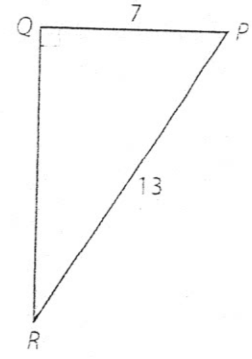
Given an acute angle, $\angle A$,

- if $\sin A = x$, then $\sin^{-1} x = m\angle A$, read as "inverse sine of x"
- if $\cos A = x$, then $\cos^{-1} x = m\angle A$, read as "inverse cosine of x"

You can use a calculator to evaluate inverse trigonometric expressions.

To use the inverse function for any trig function, first hit the 2nd button. Then hit the function you wish to use. This puts it into Inverse mode.

Example 4 Find the acute angle measures in $\triangle PQR$, to the nearest degree.



(A) Write a trigonometric ratio for $\angle R$.

Since the lengths of the hypotenuse and the opposite leg are given, use the sine ratio. $\sin R = \frac{PQ}{PR}$

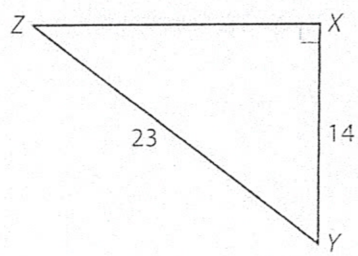
Substitute 7 for PQ and 13 for PR. $\sin R = \frac{7}{13}$

(B) Write and evaluate an inverse trigonometric ratio to find m∠R and m∠P.

Start with the trigonometric ratio for ∠R.	$\sin R = \frac{7}{13}$
Use the definition of the inverse sine ratio.	$m\angle R = \sin^{-1} \frac{7}{13}$
Use a calculator to evaluate the inverse sine ratio.	$m\angle R = 33°$
Write a cosine ratio for ∠P.	$\cos P = \frac{PQ}{PR}$
Substitute 7 for PQ and 13 for PR.	$\cos P = \frac{7}{13}$
Use the definition of the inverse cosine ratio.	$m\angle P = \cos^{-1} \frac{7}{13}$
Use a calculator to evaluate the inverse cosine ratio.	$m\angle P = 57°$

Your Turn

Find the acute angle measures in △XYZ, to the nearest degree.



13. m∠Y

$\cos Y = \frac{XY}{YZ} = \frac{14}{23}$

$m\angle Y = \cos^{-1} \frac{14}{23} \approx 53°$

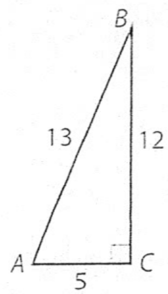
Key strokes

2nd → cos 14 ÷ 23 → Enter

14. m∠Z

Evaluate: Homework and Practice

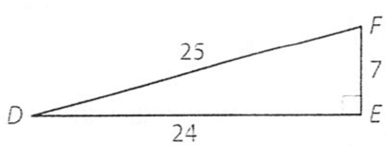
Write each trigonometric ratio as a fraction and as a decimal, rounded (if necessary) to the nearest thousandth.



3. sin A $= \dfrac{\text{opposite leg}}{\text{Hypotenuse}} = \dfrac{12}{13} \approx 0.923$

4. cos A $= \dfrac{\text{Adjacent leg}}{\text{Hypotenuse}} = \dfrac{5}{13} \approx 0.385$

5. cos B



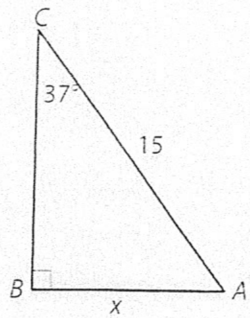
6. sin D

7. cos F

8. sin F

Find the unknown length x in each right triangle, to the nearest tenth.

9.



$\sin C = \frac{AB}{AC}$

Key strokes

15 times sin 37 enter

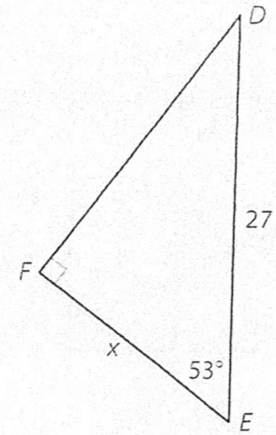
$\sin 37 = \frac{x}{15}$

$15 \cdot \sin 37 = \frac{x}{\cancel{15}} \cdot \cancel{15}$

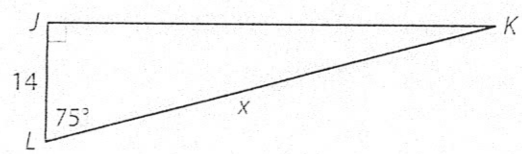
$15 \cdot \sin 37 = x$

$9 = x$

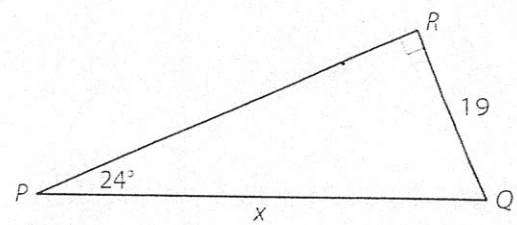
10.



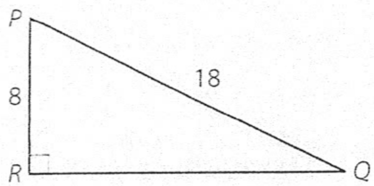
11.

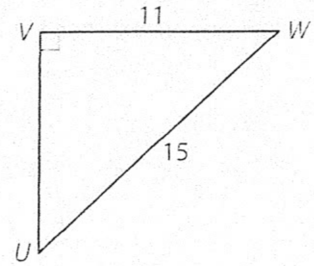


12.



Find each acute angle measure, to the nearest degree.





$\cos P = \frac{PR}{PQ}$

$m\angle P = \frac{8}{18}$

13. m∠P

$m\angle P = \cos^{-1}\frac{8}{18} = 64°$

Key strokes

2nd → cos 18 ÷ 18 → Enter

14. m∠Q

15. m∠U

16. m∠W

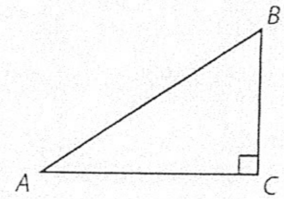
TABLE OF TRIGONOMETRIC VALUES

$m° \angle A$	sin A	cos A	tan A	$m° \angle A$	sin A	cos A	tan A
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.50	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.50	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1	90	1	0	Undefined

Name: ______________________
Date: ______________________
Period: ______________________

Geometry
Module 13 Lesson 1 K Check

For Problems 1–8, identify the features of the right triangle. The first one is done for you.



1. the hypotenuse $\overline{AB}$
2. the legs ________
3. the side opposite $\angle A$ ________
4. the side opposite $\angle B$ ________
5. the side adjacent to $\angle A$ ________
6. the side adjacent to $\angle B$ ________
7. the tangent of $\angle A$ ________
8. the tangent of $\angle B$ ________

Use a calculator to find each tangent. Round to the nearest hundredth. The first one is done for you.

13. $\tan 81° \approx$ 6.31
14. $\tan 38° \approx$ ________
15. $\tan 12° \approx$ ________
16. $\tan 30° \approx$ ________
17. $\tan 72° \approx$ ________
18. $\tan 8° \approx$ ________

The inverse tangent of *x* is the angle whose tangent is *x*. Use a calculator to find each inverse tangent. Round to the nearest 0.1 degree. Check your work by finding the tangent of each of your answers. The first one is done for you.

19. $\tan^{-1} 0.65 \approx$ 33.0°

    tan 33° ≈ 0.65

20. $\tan^{-1} \dfrac{13}{7} \approx$ ________

    tan ________ $\approx \dfrac{13}{7}$

21. $\tan^{-1} 0.4 \approx$ ________

    tan ________ ≈ 0.4

22. $\tan^{-1} \dfrac{4}{5} \approx$ ________°

    tan ________° $\approx \dfrac{4}{5}$

23. $\tan^{-1} 2 \approx$ ________°

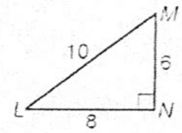
    tan ________° ≈ 2

24. $\tan^{-1} 10 \approx$ ________°

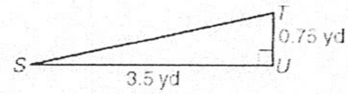
    tan ________° ≈ 10

Solve Problems 13–16 using tangent ratios and a calculator. Refer to the figure to the right of each problem.

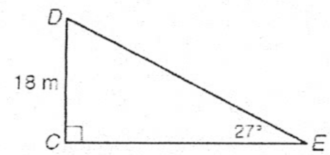
13. To the nearest hundredth, what is tan$M$ in ΔLMN? _______



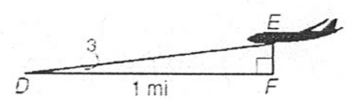
14. Write a ratio that gives tan$S$. _______ Find the value of tan$S$ to the nearest hundredth. _______ Use the inverse tangent function on your calculator to find the angle with that tangent. _______



15. Write and solve a tangent equation to find the distance from C to E to the nearest 0.1 meter. _______ meters



16. The glide slope is the path a plane uses while it is landing on a runway. The glide slope usually makes a 3° angle with the ground. A plane is on the glide slope and is 1 mile (5280 feet) from touchdown. Find EF, the plane's altitude, to the nearest foot. Show your work.



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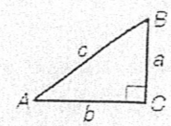
Date: ______________________
Period: ______________________

Geometry
Module 13 Lesson 2 K Check

For Problems 1–4, fill in the blanks to complete each definition. Then use side lengths from the figure to complete the trigonometric ratios. The first one is done for you.



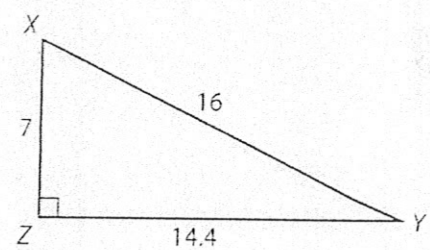
1. The sine (sin) of an angle is the ratio of the length of the leg **opposite** the angle to the length of the **hypotenuse**.

2. $\sin A = \dfrac{\square}{c}$ $\qquad \sin B = \dfrac{\square}{\square}$

3. The cosine (cos) of an angle is the ratio of the length of the leg ______________ to the angle to the length of the ______________.

4. $\cos A = \dfrac{\square}{c}$ $\qquad \cos B = \dfrac{\square}{\square}$

Use the figure to the right for Problems 5–12. Write the sines and cosines as ratios and as decimals to the nearest hundredth. Then find the measures of the angles to the nearest degree. The first one is done for you.



5. $\sin X = \dfrac{14.4}{\boxed{16}} =$ ________ 6. $\sin Y = \dfrac{\square}{\square} =$ ________

7. $\cos X = \dfrac{\square}{\square} =$ ________ 8. $\cos Y = \dfrac{\square}{\square} =$ ________

9. When you know the sine of an angle, you can find the measure of the angle in degrees by using the inverse sine, $\sin^{-1}$. Describe how to find the inverse sine of the number n on your calculator.

____

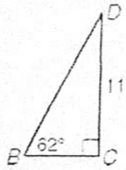
10. In Problem 5 you found the sine of $\angle X$. Use your calculator to find the inverse sine of $\angle X$, which is the measure of $\angle X$. ______________

11. Show how to use a different inverse to find $m\angle X$. (Use your answer from Problem 7.) ______________

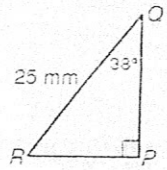
12. If you calculated $m\angle X$ correctly, what is $m\angle Y$? ____________

Confirm your answer by using the inverse cosine. ____________

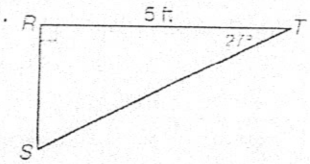
Use a calculator and trigonometric ratios to find each length. Round to the nearest hundredth.

8. 

$BD =$ ______________

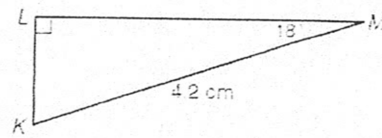
9. 

$QP =$ ______________

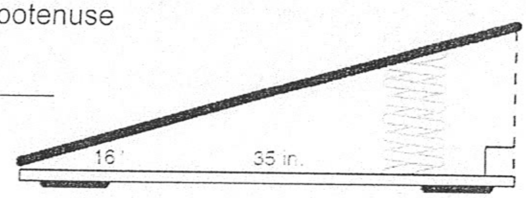
10. 

$ST =$ ______________

Use sine and cosine ratios to solve Problems 11–13.



11. Find the perimeter of the triangle. Round to the nearest 0.1 centimeter. ______________



12. To the nearest 0.1 inch, what is the length of the hypotenuse of the springboard shown to the right? ______________

13. What is the height of the springboard (the dotted line)? ______________